Journal of
Mechanical
Science and
Technology

Journal of Mechanical Science and Technology 21 (2007) 1293~1305

Numerical Investigation on the Directionality of Nonlinear Indicial Responses

Kwanjung Yee[a,*], Sangwon Hong[b], Dong-Ho Lee[b]

[a]Department of Aerospace Engineering Pusan National University, Busan 609-735, S. Korea
[b]School of Mechanical and Aerospace Engineering Seoul National University, Seoul 151-742, S. Korea

(Manuscript Received January 23, 2007; Revised April 24, 2007; Accepted April 24, 2007)

Abstract

An unsteady Euler solver is modified to investigate the directionality of nonlinear indicial response to a step change in the angle of attack. An impulsive change in the angle of attack is incorporated by using the field velocity approach, which is known to decouple the step change in the angle of attack from a pitch rate. Numerical results are thoroughly compared against analytical results for two-dimensional indicial responses. The same method is applied to investigate the directionality of nonlinear indicial responses. It is found that directionality is mainly due to the asymmetry of initial shock locations. Since the directionality of the pitching moment responses is significant in the critical Mach number region, it is also shown that consideration of the directionality is crucial for accurate modeling of the nonlinear indicial functions.

Keywords: CFD; Indicial response; Directionality; Field velocity approach; Initial shock location

1. Introduction

The accurate prediction of the aerodynamic loads on a helicopter cannot be achieved without the proper consideration of unsteady aerodynamics. Unsteady aerodynamic effects have substantial impacts on the aeroelastic behaviors of the rotor, vibratory loads and flight performance. Accordingly, the proper modeling of unsteady aerodynamic effects of the rotor system is one of the most challenging problems for the analysts. Depending on its usage, there should be some sort of compromise between accuracy and computational efficiency. Most comprehensive codes used for routine analyses and design are based on the classical unsteady approach – the indicial theory – because it enables the calculations of aerodynamic loads at relatively low computational cost. The indicial theory can provide with good efficiency the time domain unsteady aerodynamic responses to arbitrary forcing on the blade. Detailed description about the linear indicial theory can be found in Ref. (Leishman and Beddoes, 1986). The indicial method is based on the linear incompressible theory, which limits the level of confidences in the flight conditions where the nonlinear aerodynamic characteristics are dominant such as in critical Mach number and high angle of attack. The helicopter rotor environment involves compressibility effects and changes in forcing at high reduced frequencies, rendering incompressible theories essentially invalid. Hence, alternative approaches are required to extend the indicial method to introduce compressibility.

Recently, Lee (2003) explored the limits of linear indicial theory and developed a nonlinear indicial response method for unsteady airloads predictions. They extracted the nonlinear indicial responses from CFD calculations and represented them in functional forms. The indicial response functions were then used

*Corresponding author. Tel.: +82 51 510 2481, Fax.: +82 51 513 3760
E-mail address: daedalus@pusan.ac.kr

with Duhamel superposition to find the unsteady airloads under arbitrary conditions. It has been found that the onset of supercritical flow introduces strong nonlinearities in the flow fields the nonlinear indicial method yields more accurate results than the linear indicial method. Directionality behavior was also examined during the process of identifying the nonlinear indicial response functions. It was found that at subcritical Mach number, directionality issues were negligible however, at critical Mach number, large differences were noted in the indicial responses depending on direction of the step input. They concluded that the directionality issues are of secondary importance because the discrepancies in the indicial responses were confined only to an early stage. However, the origin of the directionality and the flow regime where directionality is negligible are not known yet.

One major drawback of the indicial method is due to the fact that the indicial response is a mathematical concept and cannot be determined by direct experiment. For inviscid and incompressible flow, exact closed-form analytical solutions exist. However, for compressible flow, no exact closed-form analytical solutions are available. Although Lomax (1960) derived exact analytical results of the indicial responses to a step change in angle of attack, in pitch rate, and for the a sharp-edged gust in subsonic compressible flow, they are known only for a short period of time after the step input is applied. As in Leishman's work (1993), indicial lift response can be obtained from the frequency domain measurements. However, computational fluid dynamics (CFD) is more widely used recently in determining the indicial responses for various flow regimes (Lee et al., 2003; Singh, 1997). CFD yields reliable results from the comparison of reliable experiment or analytical results. CFD has another advantage in that it can provide the time history of the pressure distributions around the airfoil, so it enables a detailed physical investigation of the flow field. In the present work, an unsteady Euler solver is modified to investigate the indicial responses to a step change in angle of attack. The incorporation of a step change into CFD is performed by using field velocity approach suggested by Baeder (1997). The field velocity approach leads to a natural decoupling of input parameters such as angle of attack and pitch rate, and this decoupling is crucial in determining the indicial responses. There are theoretically no different indicial responses between the change angle of attack from 2° to 4° and the change angle of attack from 4° to 2°, in a fixed Mach number, but in actually there are some di-fferences because of the shock in transonic region and so on. This specific character of the indicial responses is labeled as directionality.

The principal aim of the present work is twofold. One is to develop a reliable computational tool to obtain linear and nonlinear indicial responses and their functional forms. To this end, an Euler solver is modified by using the field velocity approach, by which the step input is incorporated in the CFD code. Validation has been made by comparing the computed results with the exact closed-form analytical solutions.

Next, it is intended to show that the consideration of the directionality is essential when modeling the indicial response functions at the critical Mach number. First, the indicial responses are obtained for different mean angles of attack and step inputs and the origin of the directional asymmetry is discussed. It is found that the directionality due to the nonlinearity of the flow fields is not negligible for most critical Mach numbers and asymmetry remains for a long time after the step input is applied ($s > 70$).

2. Numerical approach

2.1 Governing equations

Two-dimensional unsteady Euler equations are used for this study. By introducing the general curvilinear coordinate system $\xi=\xi(t, x, y)$ and $\eta=\eta(t, x, y)$, the governing equations are written in conservative form.

$$\frac{1}{J}\frac{\partial Q}{\partial t_c}+\frac{\partial E}{\partial \xi}+\frac{\partial F}{\partial \eta}=0 \qquad (1)$$

In Eq. (1), Q represents the conservative variables $[\rho, \rho u, \rho v, \rho e]^T$ in a two-dimensional problem. E and F are the convection terms. t_c is the computational time and t is physical time and $J = \xi_x\eta_y-\eta_x\xi_y$ stands for the transformation Jacobian. All geometric variables are non-dimensionalized by the airfoil chord length c, density ρ by the free stream density ρ_∞, and velocity u, v by speed of sound a_∞, separately.

Dual time stepping method was used to obtain higher order temporal accuracy for unsteady flow fields. Roe's flux difference scheme is used for spatial discretization of inviscid terms. Computation time has been greatly reduced by parallel computing for which LU-SGS (Lower-Upper Symmetric Gauss-Seidel) method is employed.

For steady calculations, convergence criteria were set to 10^{-6} based on the residual of density. A 259x61 C-type grid with 159 points on the airfoil surface was used for the baseline of a clean airfoil. C-type 127×45, 259×61 and 323×91 grids are used for spatial convergence check and physical time step is varied with 0.1022, 0.0510 (single period divided by 1440 physical time step), and 0.0255 is used for temporal accuracy check. As a result of these convergence tests, convergence criteria were well matched to 10^{-6} and there are no significant differences in these tests. Therefore, the results of these tests are not shown in this study.

2.2 Field velocity approach

It is necessary to incorporate a step change into the CFD codes for direct indicial response calculations. However, this is quite challenging since the step change in one of the parameters – e.g., angle of attack – may change another input so that the output may not be the correct indicial response. For example, if the airfoil is suddenly exposed to the flow at a different angle of attack, then the airfoil will also experience an infinite pitch rate. Then, the response will certainly be a reflection of a step change in the angle of attack as well as an impulsive change in the pitch rate.

In order to overcome the coupling issue, the field velocity approach is used in the present unsteady solvers. In the field velocity approach, the unsteady flow is modeled via grid movement. The basic idea is to incorporate the step change in the input as a step change in the grid velocity over the entire flow domain. For example, a step change in the angle of attack is incorporated as a step change in the vertical velocity over the all flow domain, which is equivalent to a step change in the angle of attack. It should be noted that there is no coupling influence on pitch rate because the airfoil does not undergo any pitching motion. Hence, the indicial response of a step change in the pitch rate is decoupled from that in the angle of attack. As will be shown later, the approach is found to be numerically robust and yields reliable results compared with exact analytic solutions.

Let the induced velocity by the vortex wake be represented by the velocity field (u', v', w'). Then the resultant velocity field becomes Sitarman and Baeder (2004).

$$\vec{V} = (u - x_\tau + u')\hat{i} + (v - y_\tau + v')\hat{j} + (w - z_\tau + w')\hat{k} \tag{2}$$

The field velocity approach models this changed velocity field by changing the grid velocities. The modified grid velocities are defined as

$$\tilde{x}_\tau \hat{i} + \tilde{y}_\tau \hat{j} + \tilde{w}_\tau \hat{k} = (x_\tau - u')\hat{i} + (y_\tau - v')\hat{j} + (z_\tau - w')\hat{k} \tag{3}$$

2.3 Indicial Method

The indicial approach is based on the concept that an aerodynamic response $f(t)$ can be linearized with respect to its forcing function $\varepsilon(t)$, but only if $f(t)$ is a smooth function of $\varepsilon(t)$. This allows the representation of $f(t)$ in a Taylor series about some value of $\varepsilon = \varepsilon_0$, i. e. Leishman (2006),

$$f(t) = f(0) + \Delta\varepsilon \left.\frac{\partial f}{\partial \varepsilon}\right|_{\varepsilon=\varepsilon_0} + \cdots \tag{4}$$

If the response $\partial f / \partial \varepsilon$ depends only on the elapsed time from the perturbation $\Delta\varepsilon$, then it may be shown that the formal solution for $f(t)$ is the well-known Duhamel integral

$$f(t) = f(0) + \int_0^t \frac{d\varepsilon}{d\sigma}\phi(t-\sigma)d\sigma \tag{5}$$

where $\phi(t) = \partial f / \partial \varepsilon|_{\varepsilon=\varepsilon 0}$.

Hence, if the forcing function ε is known and if $\phi(t)$ is also known, then solving the Duhamel integral gives a solution to $f(t)$ for any arbitrary changes in $\varepsilon(t)$.

Approximate functional representations of a two-dimensional response are in general represented as a summation of exponential functions which decay with nondimensional time (s). Non-dimensional time s is defined as the distance traveled in semi-chords by the airfoil from the instant of the indicial change $(s = 2U_\infty t / c)$.

The aerodynamic response to changes in the angle of attack and pitch rate consists of circulatory and noncirculatory parts. The circulatory part is associated with the formation of circulation around the airfoil, and the noncirculatory part is associated with apparent mass effects.

The functional forms of the lift and pitching moment response for a step change in the angle of attack and pitch rate obtained by Leishman (2006) are given as follows

$$\frac{C_{n_\alpha}(s)}{\alpha}=\frac{4}{M}\phi_\alpha^{nc}(s,M)+C_{n_\alpha}\phi_\alpha^{c}(s,M) \tag{6}$$

$$\frac{C_{m_\alpha}(s)}{\alpha}=-\frac{1}{M}\phi_{\alpha_m}^{nc}(s,M)+C_{n_\alpha}\phi_\alpha^{c}(s,M)(0.25-x_{ac}) \tag{7}$$

$$\frac{C_{n_q}(s)}{q}=\frac{1}{M}\phi_q^{nc}(s,M)+2C_{n_\alpha}\phi_q^{c}(s,M) \tag{8}$$

$$\frac{C_{m_q}(s)}{q}=-\frac{7}{12M}\phi_{q_m}^{nc}(s,M)-\frac{C_{n_\alpha}}{4}\phi_{q_m}^{c}(s,M) \tag{9}$$

In these equations, C_{n_α} is a slope of normal force coefficient versus α curve and for incompressible flow, $C_{n_\alpha}=\frac{2\pi}{\beta}$, where $\beta=\sqrt{1-M^2}$. The subscript of these equations α, means the components resulting from angle of attack and q means the airfoil nondimensional pitch rate. And the superscripts of above equations, nc and c, mean the noncirculatory part and the circulatory part.

The functional forms and coefficients of the circulatory and non-circulatory indicial response functions were obtained by Leishman and are given as follows. The circulatory lift functions $\phi_\alpha^c(s,M)$ take the form

$$\phi_\alpha^c(s,M)=1-\sum_{i=1}^{N}A_i e^{-b_i\beta^2 s} \tag{10}$$

A two term exponential function is used to represent the indicial response as in Leishman. The non-circulatory lift functions $\phi_\alpha^{nc}(s,M)$ is given as

$$\phi_\alpha^{nc}(s,M)=e^{-s/T_\alpha} \tag{11}$$

where T_α is a time constant of decay of the non-circulatory loading. The coefficients A_i and b_i can be determined by optimization method Leishman (2006).

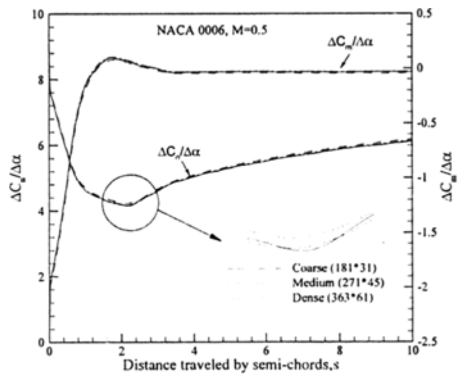
3. Results and discussion

3.1 Spatial and temporal sensitivity study

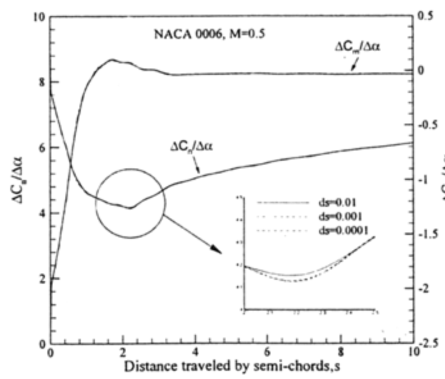
To validate the suitability of the present numerical approach, the two-dimensional indicial response of NACA 0006 airfoil to a step change in the angle of attack is numerically obtained. Unsteady computations are performed in response to 1 and 2 degree step incidence using the steady data as initial conditions.

A grid sensitivity study was performed to obtain the unsteady indicial responses for the NACA 0006 airfoil. The effect of grid density was investigated for different chordwise as well as normal directions. Three different grids - 181×31, 271×45 and 363×61 were used. Grid points were clustered properly to capture the high gradient around the stagnation point and trailing edge. In all grids, the far boundary was extended to at least 10 chord lengths from the airfoil center.

Figure 1 shows no significant change in the indicial



(a) Grid Density



(b) Time step size

Fig. 1. Influence of Grid Density and Time Step. (NACA 0006 airfoil, $M_\infty=0.5, \alpha_m=0°, \Delta\alpha=2°$)

response due to the grid density. There is a slight discrepancy in the pitching moment around $s=2$, but the estimated error is within 1%. The results suggest that a coarse grid with 181 x 31 points is sufficient for the current calculations. All the computational results shown hereafter are obtained on the coarse grid.

The sensitivity to the size of the time step was also studied for three different time steps ($ds = 0.01, 0.001, 0.0001$). As in the grid sensitivity study, the computed results are found to be insensitive to the time steps. It is concluded that the level of error due to time steps is negligible so all the computations hereafter are performed for the coarsest time step, $ds = 0.01$.

3.2 Code validation

An exact closed-form expression of the initial aerodynamic response of a flat plate can be determined by using piston theory derived by Lomax, in which the changes in lift and pitching moment are given as Manglano-Villamartin (2005),

$$\frac{\Delta C_N(s)}{\Delta\alpha} = \frac{4}{M}\left(1 - \frac{1-M}{2M}s\right)$$
$$\frac{\Delta C_m(s)}{\Delta\alpha} = -\frac{1}{M}\left(1 - \frac{1-M}{2M}s + \frac{M-2}{4M}s^2\right) \tag{12}$$

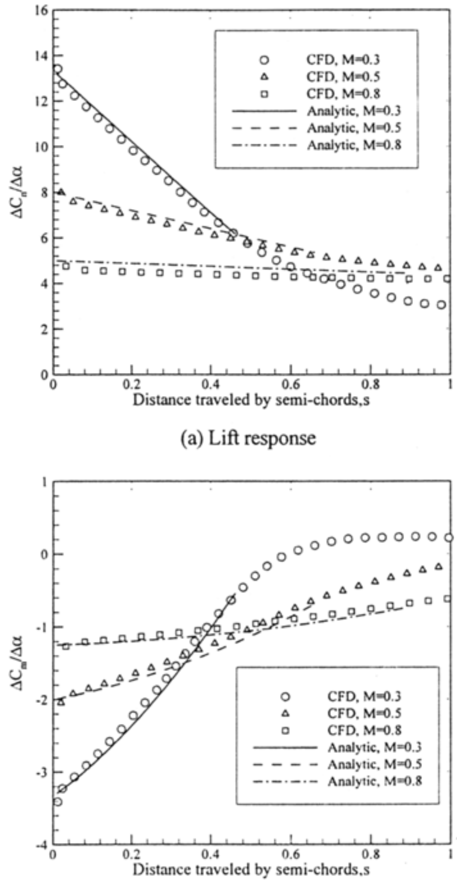
And the change of pressure distribution due to step change in the angle of attack is also given as,

$$\frac{\Delta C_p(x,\hat{t})}{\Delta\alpha} = \Re\left\{\frac{8}{\pi(1+M)}\sqrt{\frac{\hat{t}-x'}{M\hat{t}+x'}} + \frac{4}{\pi M}\left[\cos^{-1}\left(\frac{\hat{t}(1+M-2)c-x'}{\hat{t}(1-M)}\right) - \cos^{-1}\left(\frac{2x'-\hat{t}(1-M)}{\hat{t}(1+M)}\right)\right]\right\} \tag{13}$$

with $x' = x - M\hat{t}$ and $\hat{t} = s/2M$ for the period of time $0 < s \le 2M/(1+M)$, and $\Re$ means the real part where the real parts of the arc cosine of numbers greater than 1 and less than 1 are 0 and π, respectively.

Figure 2 shows the comparison of the computed and exact analytic results for a small period of time for $M_\infty = 0.3$, 0.5 and 0.8.

As shown in the figure, the computed results are in good agreement with the analytic results. However, a small discrepancy was found in the case of the higher Mach number. This is reasonable in that the piston theory is based on the linear theory. Hence, it is expected that the agreement will be worse where the nonlinear effects become dominant.

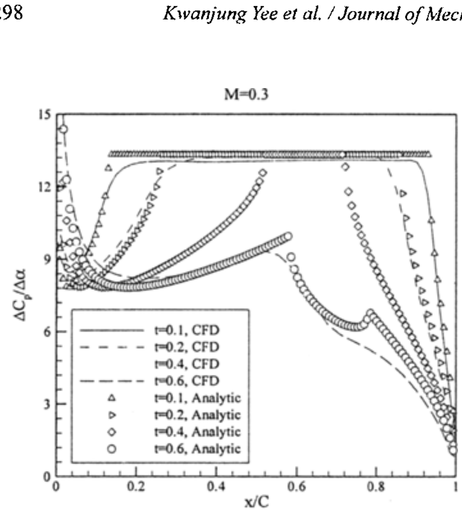


(a) Lift response

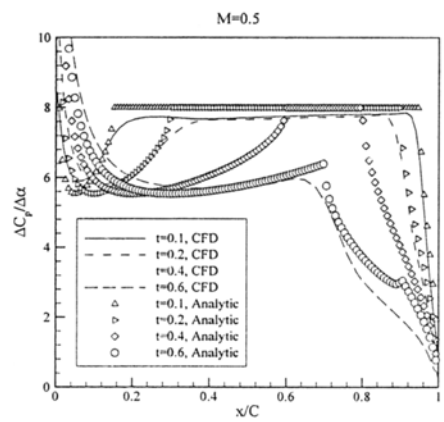
(b) Pitching moment response

Fig. 2. Comparison of CFD results for a step change in angle of attack with exact linear theory.
for $0 \le s \le 2M/(1+M)$ at $M_\infty = 0.3, 0.5, 0.8$

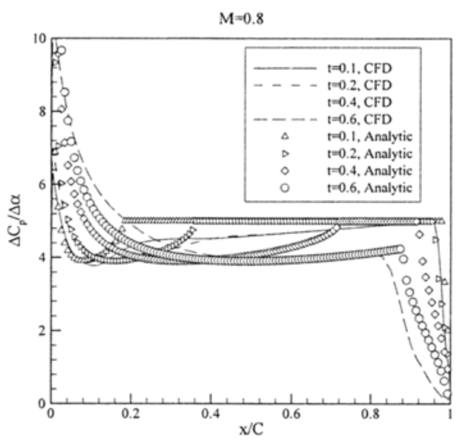
The development of pressure response to step input in the angle of attack is shown in Fig. 3. The analytic solutions from linear compressible theory are also plotted. Unlike the aerodynamic coefficients such as lift and pitching moment, the chordwise pressure distribution has inherently local and temporal nature, so that it can be a good validation for the numerical code accuracy. The exact analytic solutions are available only for a short period of time after the step input due to the aerodynamic interaction of the forward moving waves and the backward moving



(a) $M_\infty = 0.3$



(b) $M_\infty = 0.5$



(c) $M_\infty = 0.8$

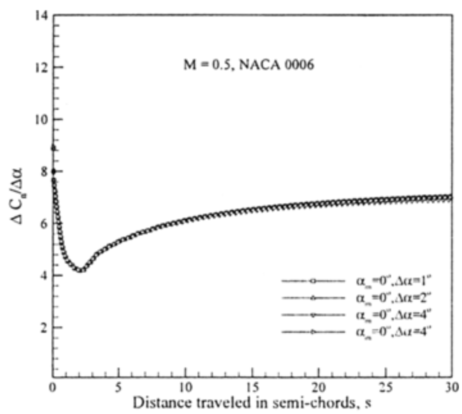
Fig. 3. Comparison of surface pressure responses from CFD results with exact linear theory for a step change in angle of attack.

waves. The comparison is made for $M_\infty = 0.3$, 0.5 and 0.8. Excellent agreement between the computed and the analytical results is found for all cases. The propagation of the pressure waves from the leading edge and the trailing edge are captured with good accuracy. However, as expected for higher Mach number, 0.8, the agreement with analytic results becomes worse due the neglect of the transonic effects of the piston theory as well as the thickness effect of the airfoil.

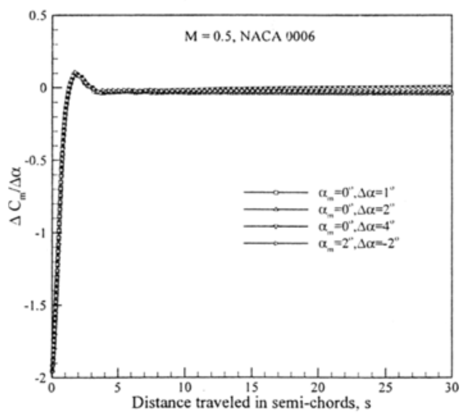
Overall, it is concluded that the numerical results are found to be reliable enough for further study.

3.3 Directionality in subcritical Mach number

Figure 4 shows the normalized indicial responses at the subcritical Mach number, 0.5 for the NACA 0006



(a) Lift response



(b) Pitching moment response

Fig. 4. Normalized indicial responses calculated by CFD for different step changes in angle of attack at $M_\infty = 0.5$.

airfoil undergoing different step inputs in the angles of attack. The computed responses are scaled to any angle of attack and to any magnitude of the step input. The scaled responses are shown as a function of the nondimensional time, s.

There is an initial transient behavior in lift, followed by an initial decay and then a more gradual build up. Shortly after indicial motions are applied, there is no circulation about the airfoil and the airloads are entirely from noncirculatory origin. During the intermediate time between the initial noncirculatory loading and the final circulatory loading, the flow adjustments are very complex. This flow involves the propagation and reflection of pressure wave-like disturbances, and the simultaneous creation of circulation about the airfoil.

It is clearly seen that the magnitudes of the step inputs have little influence on the lift and pitching moment responses. This indicates that the basic assumption of the linear indicial theory is valid at this subcritical region. The directional behavior of the indicial responses can be investigated by applying positive and negative steps and comparing the equivalent normalized indicial responses.

Notice that the indicial responses of $\alpha_m = 0°$, $\Delta\alpha = 2°$ and $\alpha_m = 2°$, $\Delta\alpha = -2°$ collapse on each other, indicating that the directionality is negligible at this speed region. For a more rigorous investigation of directionality, the chordwise pressure distribution and the development of the flow after the indicial input are shown in Figs. 5 and 6. Figure 5 shows that the CFD can capture the progression of the pressure waves from the leading edge and the trailing edge. The chordwise pressure distributions among the cases are in good agreement. Note that the pressure around the leading edge increases with time, while the pressure at the trailing edge decreases with time. The pressure distributions are shown to be smooth and no discontinuity is observed. Perturbation pressure contours in the flowfield are shown in Fig. 6 for $M = 0.3$. The perturbation pressures are obtained by subtracting the steady state pressure from the present pressure waves.

Overall, it is concluded that at subcritical Mach numbers, directionality issues are negligible.

3.4 Directionality at critical Mach number

As mentioned earlier, the indicial functions based on the linear piston theory produce accurate results for most unsteady problems. However, in the cases where the nonlinearity of the flow fields becomes dominant - shock wave or separation - the linearity assumption no longer holds. Lee et al. (2003) showed that nonlinear CFD indicial lift results at different mean angles of attack α_m, and different magnitudes of the step input $\Delta\alpha$ show significant discrepancies, which lead the conclusion that the nonlinear indicial responses cannot be easily generalized in functional form. They also noticed that in all cases considered the integrated values of airloads in the short time $(s \leq 5)$ - noncirculatory part - are linear and it is only at later times $(s \geq 10)$ – circulatory part – does this behavior becomes nonlinear. By comparing the indicial responses of positive and negative step inputs, it was concluded that directionality effects seem to be of second order in the practical sense.

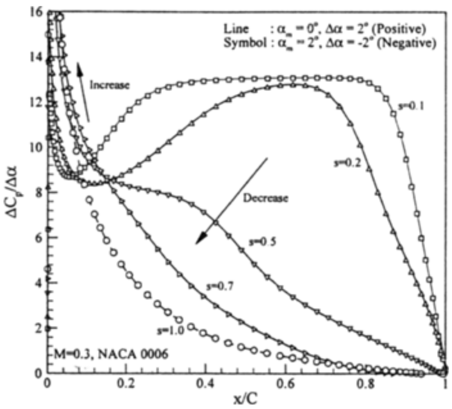


Fig. 5. Temporal progress of chordwise pressure distribution for $0 \leq s \leq 1$ at $M_\infty = 0.3$.

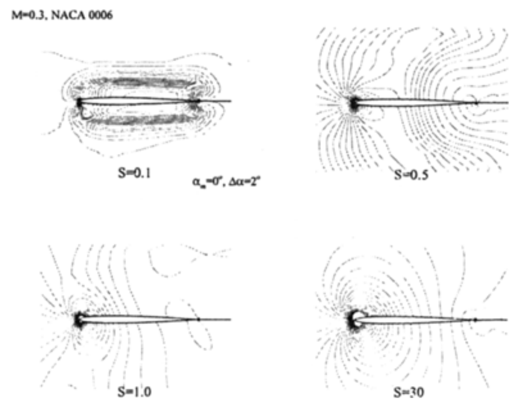


Fig. 6. Perturbed pressure contour for step input in angle of attack at $M_\infty = 0.3$.

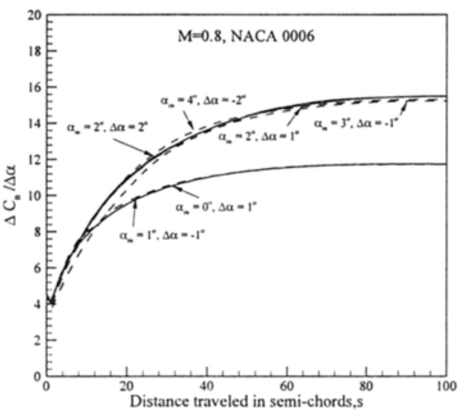
However, it seems that a more comprehensive research is required in order to draw some general conclusions about the origin and trends of directionality. In this study, directionality in the nonlinear indicial responses is investigated in more detail by using numerical analyses. To this end, a series of computations are performed by applying positive and negative steps at various mean angles of attack.

First, the normalized aerodynamic responses are shown in Fig. 7 for three different cases in order to clarify the existence of directionality at the critical Mach number region.

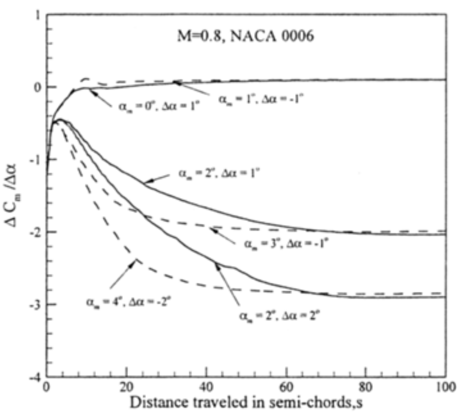
Case (1) $\alpha_m = 0^o$, $\Delta\alpha = 1^o$ vs. $\alpha_m = 1^o$, $\Delta\alpha = -1^o$

Case (2) $\alpha_m = 2^o$, $\Delta\alpha = 1^o$ vs. $\alpha_m = 3^o$, $\Delta\alpha = -1^o$

Case (3) $\alpha_m = 2^o$, $\Delta\alpha = 2^o$ vs. $\alpha_m = 4^o$, $\Delta\alpha = -2^o$



(a) Lift response



(b) Pitching moment response

Fig. 7. Normalized indicial responses for different mean angle and step changes in angle of attack at $M_\infty = 0.8$.

The first thing to note in this figure is that the indicial responses show different behavior unlike those at the subcritical Mach number. It is easily found that the indicial responses have a strong dependency on the initial mean angle of attack and the magnitude of the step input. This indicates that the validity of the linear indicial theory does not hold at this critical Mach number, which leads to the conclusion that the indicial response functions should be modified as functions of the mean angle of attack and step input for correct inclusion in the comprehensive codes.

It is obviously shown that the directionality is not significant in the lift indicial responses but is only slightly dependent on the mean angles of attack α_m and the magnitude of step input $\Delta\alpha$. On the contrary, one may notice that the pitching moment indicial responses have substantial directionality in Cases 2 and 3.

For Case 1 where α_m and $\Delta\alpha$ are relatively small, the directionality in pitching moment responses is not apparent except at the noncirculatory region. However, in Cases 2 and 3, the pitching moment indicial responses show considerably different behavior with time depending on the direction of step input. The normalized indicial responses for pitching moment persist even until s=70 in Case 2. Note that the discrepancy is significant even around the noncirculatory region. In the frequency domain, the noncirculatory region corresponds to the high frequency region. Hence, the computed results indicate that the nonlinear indicial response functions which do not consider directionality effects would yield inaccurate solutions for cases such as the cases of the airfoils oscillating at high frequency. A more detailed discussion will be addressed in a separate paper.

Figure 8 shows the perturbed surface pressure contours from $s = 0.1$ to $s = 10$ for Case 2. The perturbed surface pressure contours in the flow field are obtained by subtracting the steady state pressure from the current pressure to remove the thickness effects. It is clearly illustrated that the pressure disturbance builds up and propagates as time progresses. Unlike in the subcritical Mach number, distinct asymmetry exists in the pressure distribution depending on the initial mean angle of attack and the direction of the step input.

In general, the shock location on the airfoil moves backwards with the increase of the angle of attack.

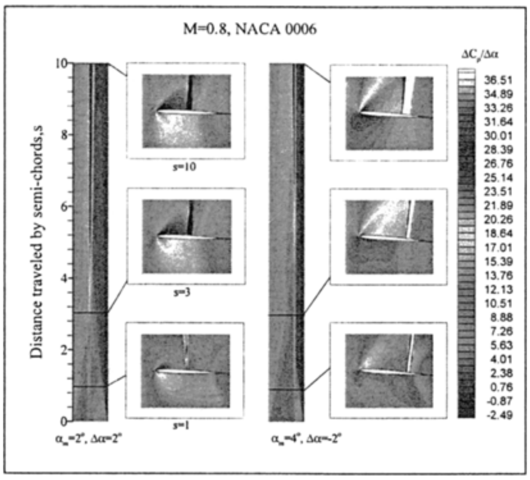
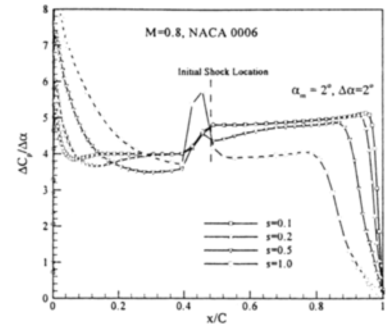


Fig. 8. Chordwise pressure difference contour with time, s for different mean angle and step size
(left : positive $\alpha_m = 2^\circ, \Delta\alpha = 2^\circ$, right : negative $\alpha_m = 4^\circ$, $\Delta\alpha = -2^\circ$).

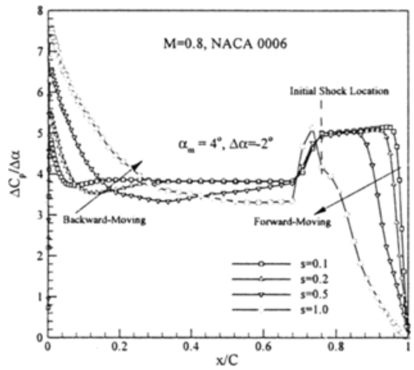
For a positive step input ($\alpha_m = 2^\circ$ and $\Delta\alpha = 2^\circ$), the shock stands at around mid-chord. The initial shock location for a negative step input ($\alpha_m = 4^\circ$ and $\Delta\alpha = -2^\circ$) is about 3 quarter chord. The difference in the initial shock location persists in the ensuing pressure distribution. At $s = 5$ where the circulatory airloads begin to build up, the discontinuity of the pressure distribution is apparently visible and becomes stronger as time elapses.

In an effort to investigate the origin of directionality in the pitching moment indicial responses, the instantaneous perturbed pressure coefficients are depicted for various temporal locations as in Fig. 9. Note the perturbed pressure distributions with time in Fig. 9 in comparison with Fig. 6. It is readily seen that there exists a pressure jump on the airfoil right after the step input is applied, s=0.1. The discontinuity of the pressure distribution at the initial stage is due to the shock wave on the airfoil surface. The discontinuous pressure jump evolves with time as the initial shock moves toward the final location. The forward moving pressure wave and the aft moving pressure waves are clearly shown in Fig. 10. It can be also found from the perturbed pressure contours that the pressure distribution is significantly different from each other even at the early stage of indicial responses.

Figures 11 and 12 illustrate the temporal progress of chordwise pressure distribution and contours at the circulatory flow region. Note that the circulation



(a) $\alpha_m = 2^\circ, \Delta\alpha = 2^\circ$



(b) $\alpha_m = 4^\circ, \Delta\alpha = -2^\circ$

Fig. 9. Temporal progress of chordwise pressure distribution for $0 \le s \le 1$ at $M_\infty = 0.8$.

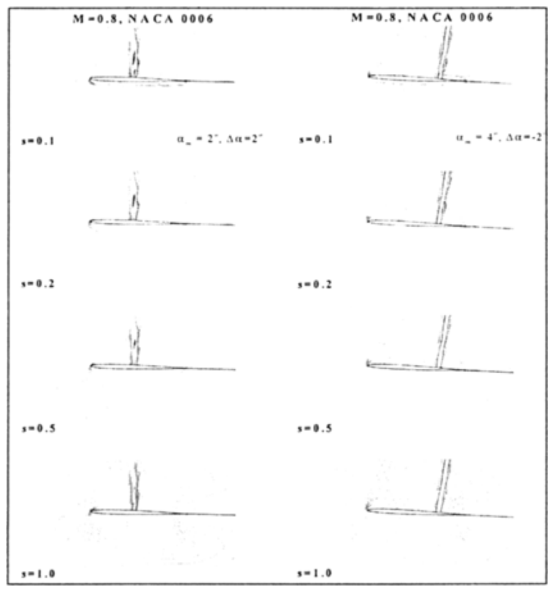
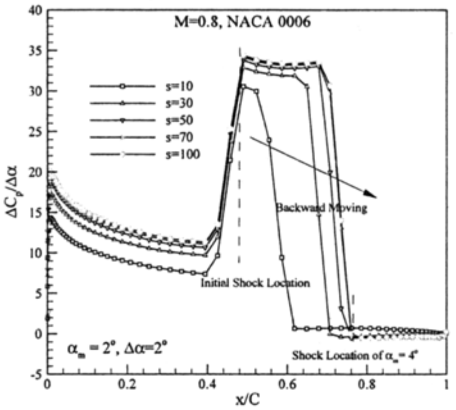
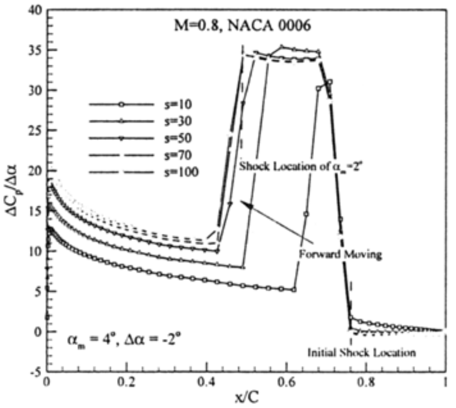


Fig. 10. Comparison of perturbed pressure contours for different direction of step input.
(left : positive $\alpha_m = 2^\circ, \Delta\alpha = 2^\circ$, right : negative $\alpha_m = 4^\circ$, $\Delta\alpha = -2^\circ$)



(a) $\alpha_m = 2°, \Delta\alpha = 2°$



(b) $\alpha_m = 4°, \Delta\alpha = -2°$

Fig. 11. Temporal progress of chordwise pressure distribution for $10 \le s \le 100$.

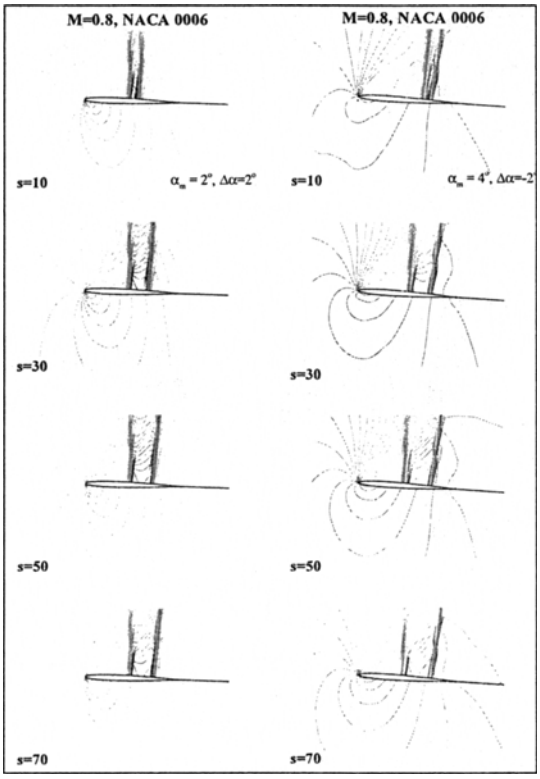


Fig. 12. Comparison of perturbed pressure contours for different direction of step input.
(left : positive $\alpha_m = 2°, \Delta\alpha = 2°$, right : negative $\alpha_m = 4°, \Delta\alpha = -2°$)

around the airfoil begins to build up and the perturbed pressure reaches its maximum at around $s = 10$. In the case of the positive step input ($\alpha_m = 2°$, $\Delta\alpha = 2°$), the shock wave moves backwards from the initial to the final position. On the contrary, in case of the negative step input the shock wave moves forwards from the initial to the final position. As expected theoretically, the final shock position of the positive step input case is almost the same as the initial position of the negative step input case. The movement of the shock wave and ensuing asymmetry of the perturbed pressure are clearly shown in Fig. 12. It is seen that the region of discontinuity in the pressure becomes wider as the time progresses and the direction of the pressure evolution is dependent on the initial mean angle of attack. It should be noted here that the difference of the pitching moment responses becomes less than 0.01 only after $s = 70$. This implies that the directionality should be of major consideration at the critical Mach number, especially for the pitching moment indicial responses.

To quantify the directionality due to shock presence at the critical Mach number, the directionality factor J is defined as follows. It is the average value for the certain period of time of the absolute difference in the pitching moments for the direction of step input.

$$J = \frac{1}{S_{\max}} \int_0^{s_{\max}} \left| \frac{\Delta C_{m+}}{\Delta\alpha} - \frac{\Delta C_{m-}}{\Delta\alpha} \right| ds \tag{14}$$

where $S_{\max}$ is set to 100 and ds is 0.2.

Figure 13 illustrates the directionality factors for various mean angles of attack α_m at which the step input is applied and the magnitudes of the step inputs. The computations are performed for 4 different mean angles of attack, $\alpha_m = 0°, 1°, 2°, 3°$ and for 4 different step inputs, $\Delta\alpha = 1°, 2°, 3°, 4°$. It can be seen that the

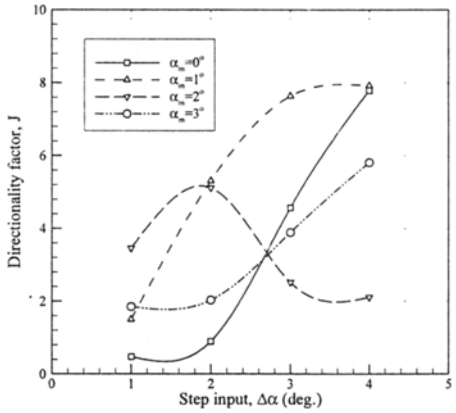
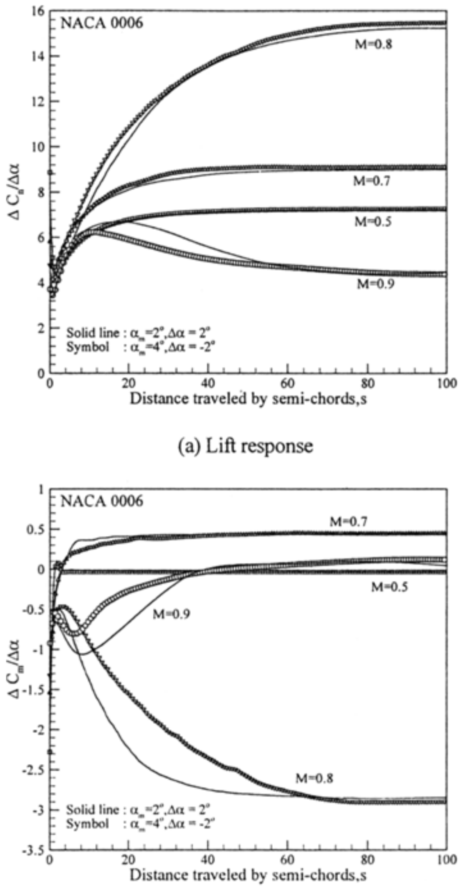


Fig. 13. Comparison of directionality factor with respect to α_m am and $\Delta\alpha$.

directionality has a general tendency to increase with the magnitude of the step input for a given angle of attack. However, for the case of $\alpha_m = 2''$, the directionality decreases after $\Delta\alpha = 2''$, the reason for which is not understood yet. The computed results indicate that the directionality of the indicial responses at the critical Mach number has an irregular and nonlinear behavior. Hence, it is expected that much effort should be taken further in order to take the directionality of the indicial responses into modern-day comprehensive codes such as CAMRAD II and FlightLab. Finally, the effects of Mach number and airfoil thickness on directionality have been also examined. The comparison of directionality factor with respect to the Mach number is made in Fig. 14. As expected, it is found that no directionality exists for both lift and pitching moment indicial responses at the subcritical Mach number. On the contrary, once the shock occurs, directionality is shown to be obvious. Intuitively, it is expected that the directionality should increase with the free stream Mach number. However, it is not the case as shown in the figure.

Figure 15 shows the influence of airfoil thickness on the directionality of indicial responses for $(t/C)_{\max} = 6$, 12 and 15% . It is found (in the figure) that the directionality decreases with the airfoil thickness. Although it seems rather paradoxical, it is due to the fact that once the shock occurs on the airfoil, the difference in the initial shock locations, which are shown to have a dominant effect on the directionality decreases with the Mach number and thickness. This implies that the initial shock locations are the most important factor of directionality.



(a) Lift response

(b) Pitching moment response

Fig. 14. Influence of Mach number on the directionality of indicial responses.

4. Concluding remarks

In the present study, an unsteady Euler solver was modified to investigate the directionality of nonlinear indicial response to a step change in the angle of attack. The pitching moment indicial responses and the pressure contours were shown for various cases to explain the origin of the directionality.

Based on the analyses of the computed results, the following conclusions have been drawn.

1. It is confirmed again in this paper that the indicial responses have highly nonlinear behavior at the critical Mach number and there exists a significant directionality with respect to the mean angles of attack and the magnitudes of the step input.

2. It has been shown that the directionality at the critical Mach number stems from the initial asymmetry of the shock locations at different mean angles of attack. The indicial lift responses are found

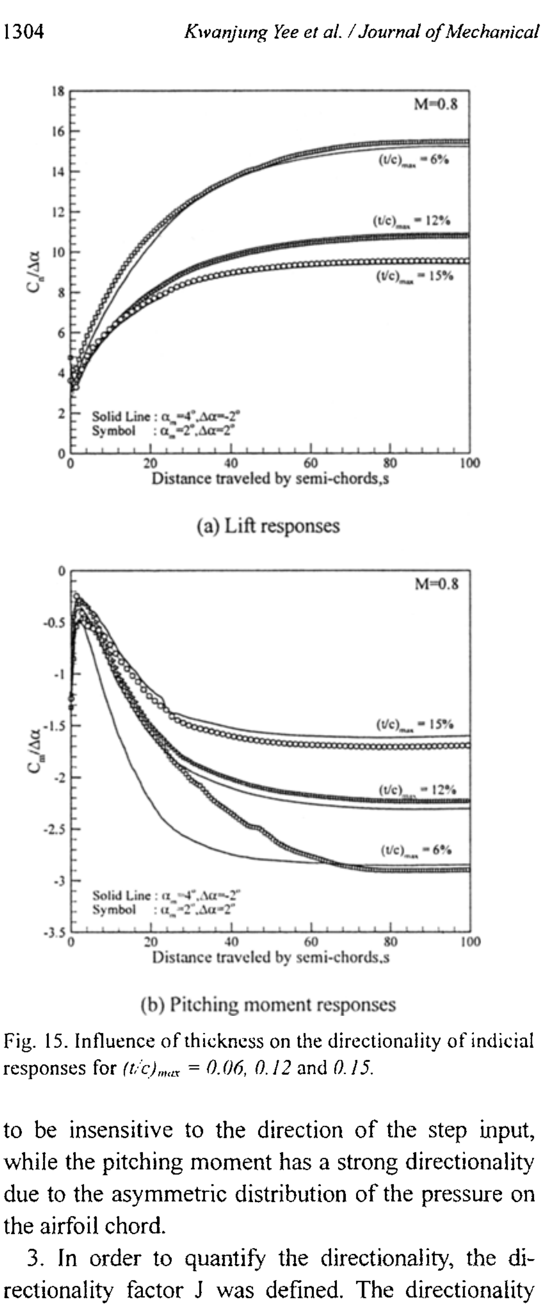


Fig. 15. Influence of thickness on the directionality of indicial responses for $(t/c)_{max}$ = 0.06, 0.12 and 0.15.

to be insensitive to the direction of the step input, while the pitching moment has a strong directionality due to the asymmetric distribution of the pressure on the airfoil chord.

3. In order to quantify the directionality, the directionality factor J was defined. The directionality factors for various mean angles of attack and step inputs were compared for M = 0.8. It is shown that the directionality tended to increase with the mean angles of attack and the step inputs. However, the behavior is highly nonlinear and no generalization was possible to be drawn.

4. The effects of Mach number and airfoil thickness on the directionality were also examined. For the same airfoil, the directionality was found to decrease with the Mach number and airfoil thickness. Although it seemed rather paradoxical, it was due to the fact that once the shock occurred on the airfoil, the difference in the initial shock locations, which were shown to dominant effect on the directionality decreased with the Mach number and thickness. This implied that the initial shock locations are the most important factor of the directionality.

Acknowledgement

This work was supported for two years by Pusan National University Research Grant and by the Brain Korea-21 Program (BK21) for the Research center of excellence focusing on the helicopter core technologies at Pusan National University. And this work was supported by the Brain Korea-21 Program (BK21) for the Mechanical and Aerospace Engineering Research at Seoul National University

Nomenclature

α : Angle of attack
α_m : Mean angle of attack
$\Delta\alpha$: Increment(decrement) of angle of attack
ξ, η : Time dependent curvilinear coordinates
ρ_∞ : Free stream density
ω : Frequency of oscillating airfoil motion (rad/sec)
a_∞ : Speed of sound
A_n : Coefficient of circulatory indicial response functions
b_n : Exponents of indicial response functions
c : Airfoil chord
C_m : Pitching moment coefficient about 0.25-chord
C_n : Normal force coefficient
$\boldsymbol{E}, \boldsymbol{F}$: Convective fluxes
J : Jacobian of transformation matrix
k : Reduced frequency = $\omega c/2V$
M_{DD} : Zero-lift drag divergence Mach number
M_∞ : Free stream Mach number
Q : Vector of conservative variables
q : Normalized pitch rate = $\dot{\alpha}c/V$
s : Distance in semi-chords = $2Vt/c$
t : Non-dimensionalized physical time
x_{ac} : Non-dimensional center of pressure

References

Lee, D. et al., 2003, "A Nonlinear Indicial Method for the Calculation of Unsteady Airloads," *59th Annual Forum and Technology Display of American Helicopter Society International*, Phoenix, AZ.

Leishman, J. G. and Beddoes, T. S., 1986, "A Generalized Model for Airfoil Unsteady Aerodynamics Behavior and Dynamic Stall Using the Indicial Method," *42nd Annual Forum of the American Helicopter Society*, Washington D.C.

Leishman, J. G, 1993, "Indicial Lift Approximations for Two-Dimensional Subsonic Flow as Obtained from Oscillatory Measurements," *Journal of Aircraft*, Vol. 30, No. 3, pp. 340~351.

Leishman, J. G, 2006, Principles of Helicopter Aerodynamics, Cambridge University Press, 2nd Edition

Lomax, H., 1960, "Indicial Aerodynamics," *AGARD Manual for Aeroelasticity*, Pt. II, Nov., Chap. 6.

Manglano-Villamarin, C. E. and Shaw, S. T., 2005, "Three-Dimensional Indicial Response of Finite Aspect Ratio Yawed Wings," *31st European Rotorcraft Forum*, Firenze, Italy.

Parameswaran, V. and Baeder, J. D., 1997, "Indicial Aerodynamics in Compressible Flow-Direct Computational Fluid Dynamic Calculations," *Journal of Aircraft*, Vol. 34, No. 1, pp. 131~133.

Singh, R., 1997, "Direct Calculation of Three-Dimensional Indicial Lift Response Using Computational Fluid Dynamics," *Journal of Aircraft*, Vol. 34, No. 4, pp. 465~471.

Sitaraman, J. and Baeder, J. D., 2004, "Computational-Fluid-Dynamics-Based Enhanced Indicial Aerodynamic Models," *Journal of Aircraft*, Vol. 41, No. 4, pp. 798~810.